Robot Manipulators

Position, Orientation and Coordinate Transformations
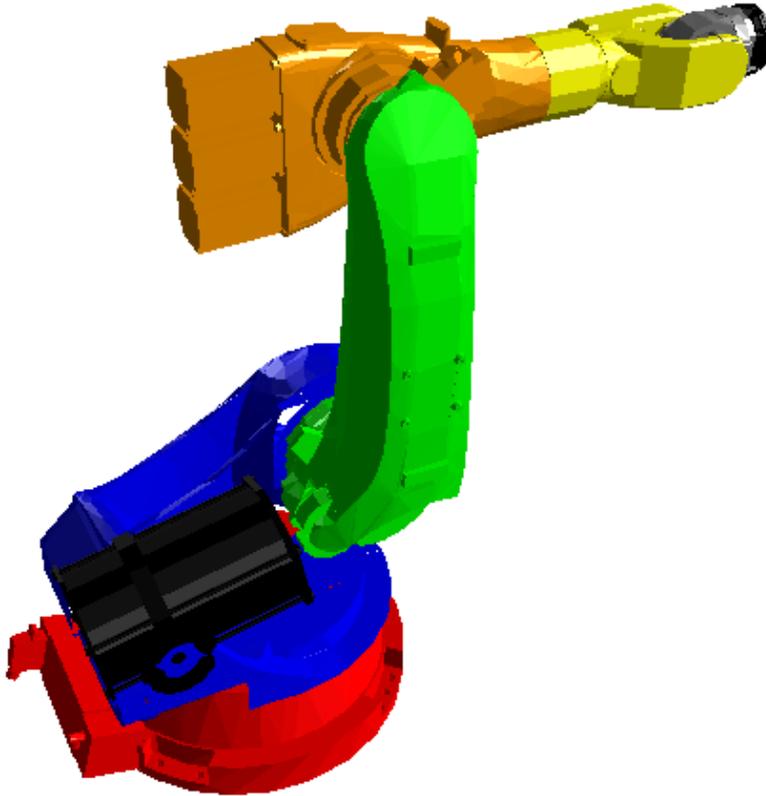
Fig. 1: Programmable Universal Manipulator Arm (PUMA)

A robot manipulator is an electronically controlled mechanism, consisting of multiple segments, that performs tasks by interacting with its environment. They are also commonly referred to as robotic arms. Robot manipulators are extensively used in the industrial manufacturing sector and also have many other specialized applications (for example, the Canadarm was used on space shuttles to manipulate payloads). The study of robot manipulators involves dealing with the positions and orientations of the several segments that make up the manipulators. This module introduces the basic concepts that are required to describe these positions and orientations of rigid bodies in space and perform coordinate transformations.

Manipulators

Manipulators are composed of an assembly of links and joints. Links are defined as the rigid sections that make up the mechanism and joints are defined as the connection between two links. The device attached to the manipulator which interacts with its environment to perform tasks is called the end-effector. In Fig. 2, link 6 is the end effector.
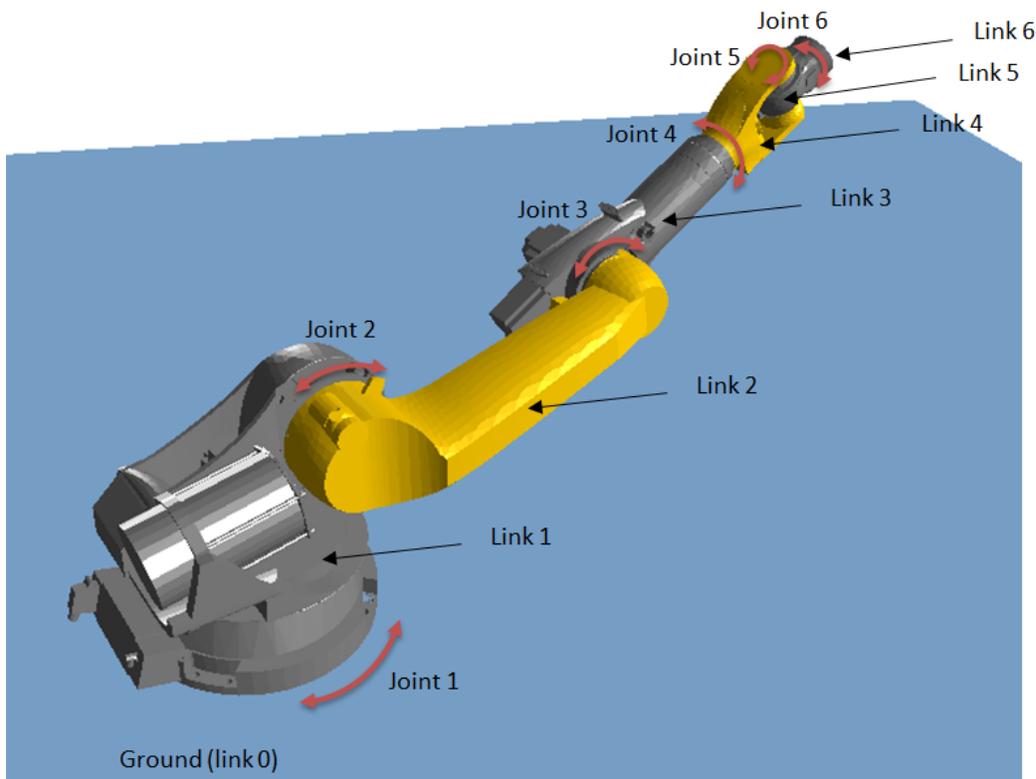
Fig. 2: Links and joints

Types of Joints

Joints allow restricted relative motion between two links. The following table describes five types of joints.

Table 1: Types of joints

Name of joint	Representation	Description
Revolute		Allows relative rotation about one axis.
Cylindrical		Allows relative rotation and translation about one axis.
Prismatic		Allows relative translation about one axis.
Spherical		Allows three degrees of rotational freedom about the center of the joint. Also known as a ball-and-socket joint.
Planar		Allows relative translation on a plane and relative rotation about an axis perpendicular to the plane.

Some Classification of Manipulators

Manipulators can be classified according to a variety of criteria. The following are two of these criteria:

By Motion Characteristics

Planar manipulator: A manipulator is called a planar manipulator if all the moving links move in planes parallel to one another.

Spherical manipulator: A manipulator is called a spherical manipulator if all the links perform spherical motions about a common stationary point.

Spatial manipulator: A manipulator is called a spatial manipulator if at least one of the links of the mechanism possesses a general spatial motion.

Open-loop manipulator (or serial robot): A manipulator is called an open-loop manipulator if its links form an open-loop chain.

Parallel manipulator: A manipulator is called a parallel manipulator if it is made up of a closed-loop chain.

Hybrid manipulator: A manipulator is called a hybrid manipulator if it consists of open loop and closed loop chains.

Degrees of Freedom

The number of degrees of freedom of a mechanism are defined as the number of independent variables that are required to completely identify its configuration in space. The number of degrees of freedom for a manipulator can be calculated as

$$n_{dof} = \lambda(n-1) - \sum_{i=1}^{k} (\lambda - f_i)$$

... Eq. (1)

where n is the number of links (this includes the ground link), k is the number of joints, f_i is the number of degrees of freedom of the ith joint and λ is 3 for planar mechanisms and 6 for spatial mechanisms.

Vector Kinematics and Coordinate Transformations

This section covers the concepts required to specify the location of an object in space. This involves specifying the position of a point on the object and the orientation of the object with respect to a reference frame.

Description of a Position

The position of any point in space, relative to a reference frame, can be described by a 3x1 position vector. For example, the position of point P (see Fig. 3) with respect to frame A can be written as

$$\vec{P}_A = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}$$

Eq. (2)

where p_x, p_y and p_z are the magnitudes of the projections of the line joining the point P and the origin O on the x, y, and z axes respectively.
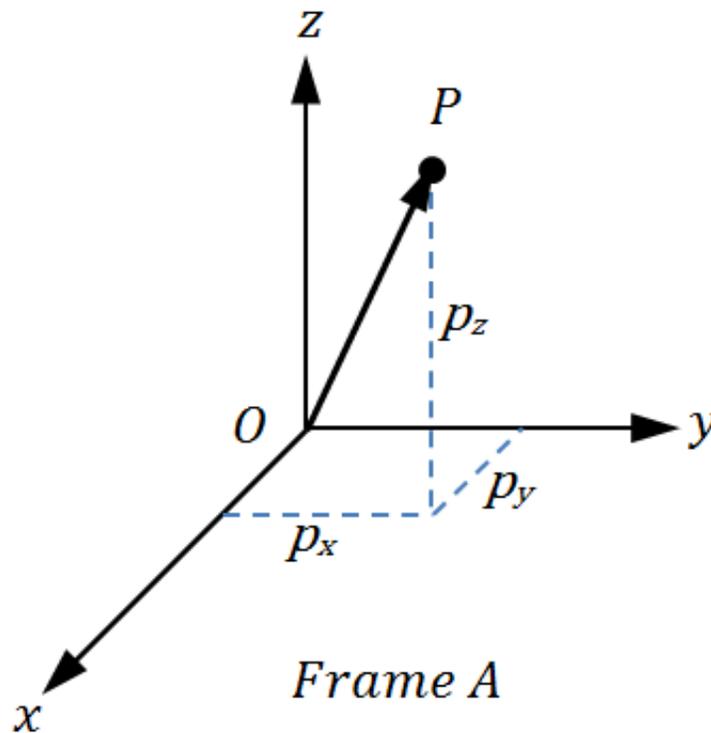
Fig. 3: Position vector

Description of an Orientation

The orientation of a body in space can be described by attaching a coordinate system to it and then describing the vectors of its coordinate axes relative to a known frame of reference. For example, the coordinate axes of Frame B (see Fig. 4) can be described relative to a known coordinate system A by the following unit vectors:

$$\widehat{x}_{AB} = \begin{bmatrix} \widehat{x}_B \cdot \widehat{x}_A \\ \widehat{x}_B \cdot \widehat{y}_A \\ \widehat{x}_B \cdot \widehat{z}_A \end{bmatrix}, \widehat{y}_{AB} = \begin{bmatrix} \widehat{y}_B \cdot \widehat{x}_A \\ \widehat{y}_B \cdot \widehat{y}_A \\ \widehat{y}_B \cdot \widehat{z}_A \end{bmatrix}, \widehat{z}_{AB} = \begin{bmatrix} \widehat{z}_B \cdot \widehat{x}_A \\ \widehat{z}_B \cdot \widehat{y}_A \\ \widehat{z}_B \cdot \widehat{z}_A \end{bmatrix}$$

... Eq. (3)

These three vectors can be combined to achieve a 3x3 matrix called a rotation matrix.

$$R_{AB} = \begin{bmatrix} \widehat{x}_B \cdot \widehat{x}_A & \widehat{y}_B \cdot \widehat{x}_A & \widehat{z}_B \cdot \widehat{x}_A \\ \widehat{x}_B \cdot \widehat{y}_A & \widehat{y}_B \cdot \widehat{y}_A & \widehat{z}_B \cdot \widehat{y}_A \\ \widehat{x}_B \cdot \widehat{z}_A & \widehat{y}_B \cdot \widehat{z}_A & \widehat{z}_B \cdot \widehat{z}_A \end{bmatrix} = \begin{bmatrix} \widehat{x}_{AB} & \widehat{y}_{AB} & \widehat{z}_{AB} \end{bmatrix}$$

... Eq. (4)
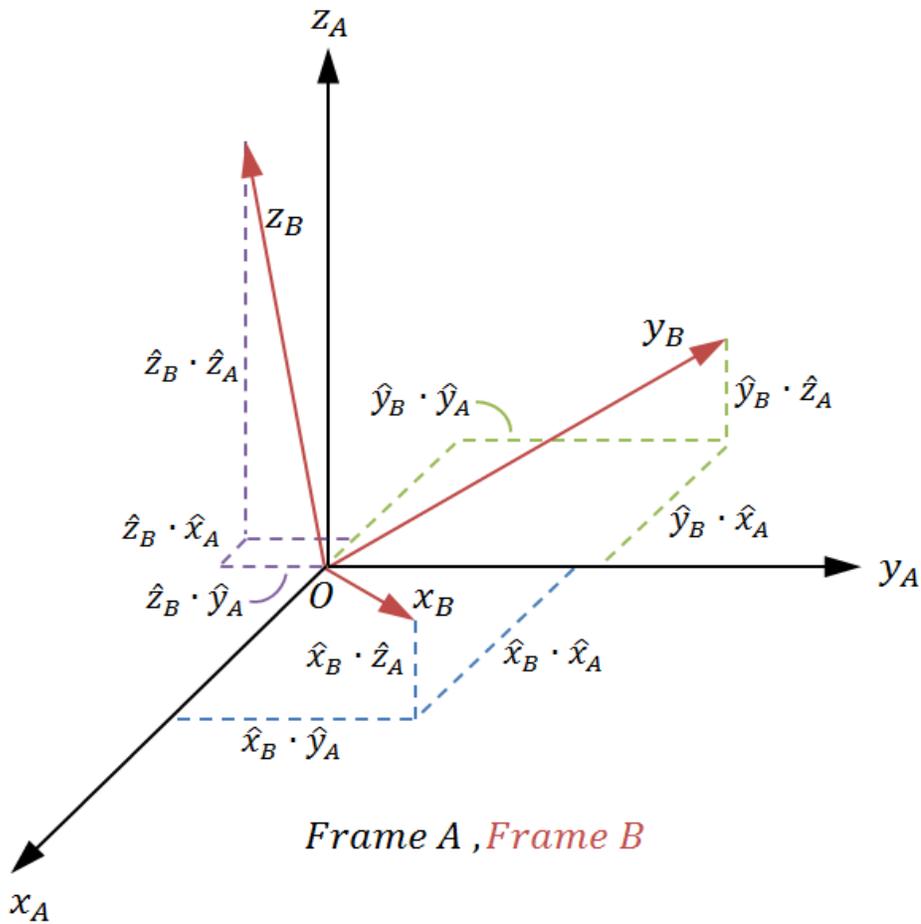
Fig. 4: Components of the rotation matrix of Frame B w.r.t Frame A

Rotation Matrix Properties

1. All the columns of a rotation matrix are orthogonal to each other.
2. The determinant of a rotation matrix is 1.
3. The inverse of a rotation matrix is equal to its transpose.

$$R_{AB} = \left(R_{BA} \right)^{-1} = \left(R_{BA} \right)^{T}$$

... Eq. (5)

This means that the rotation matrix of Frame B with respect to Frame A is equal to the

inverse and the transpose of the rotation matrix of Frame A with respect to Frame B.

Principal Rotation Matrices

Rotation about the z-axis

If a reference frame (Frame A) is rotated by an angle α about the z-axis to obtain a new frame (Frame B), the rotation matrix of the new frame is

$$R_{AB} = R_z(\alpha) = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

... Eq. (6)
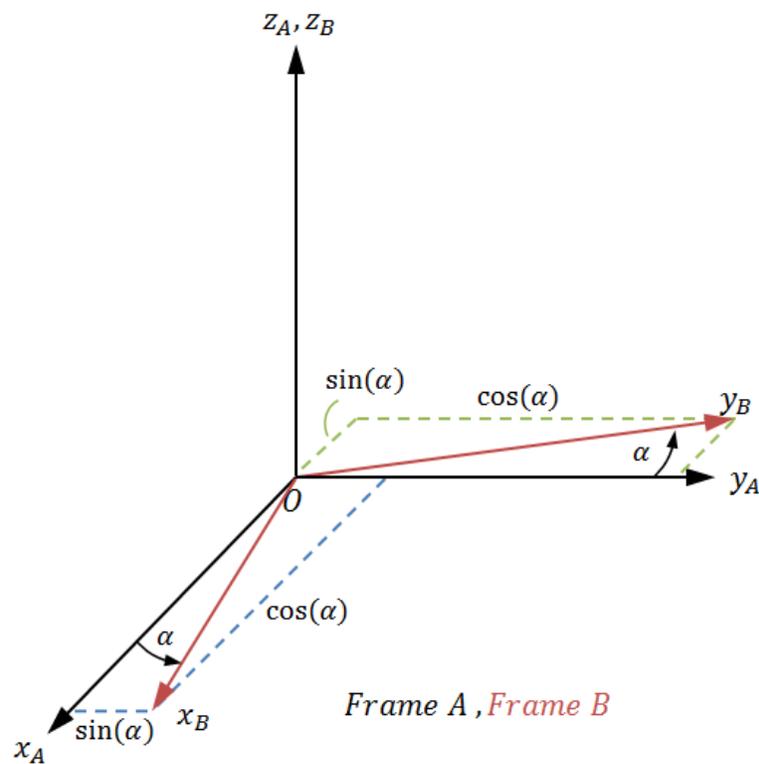
Fig. 5: Rotation about the z-axis

Rotation about the y-axis

If a reference frame (Frame A) is rotated by an angle β about the y-axis to obtain a new frame (Frame B), the rotation matrix of the new frame is

$$R_{AB} = R_y(\beta) = \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix}$$

... Eq. (7)
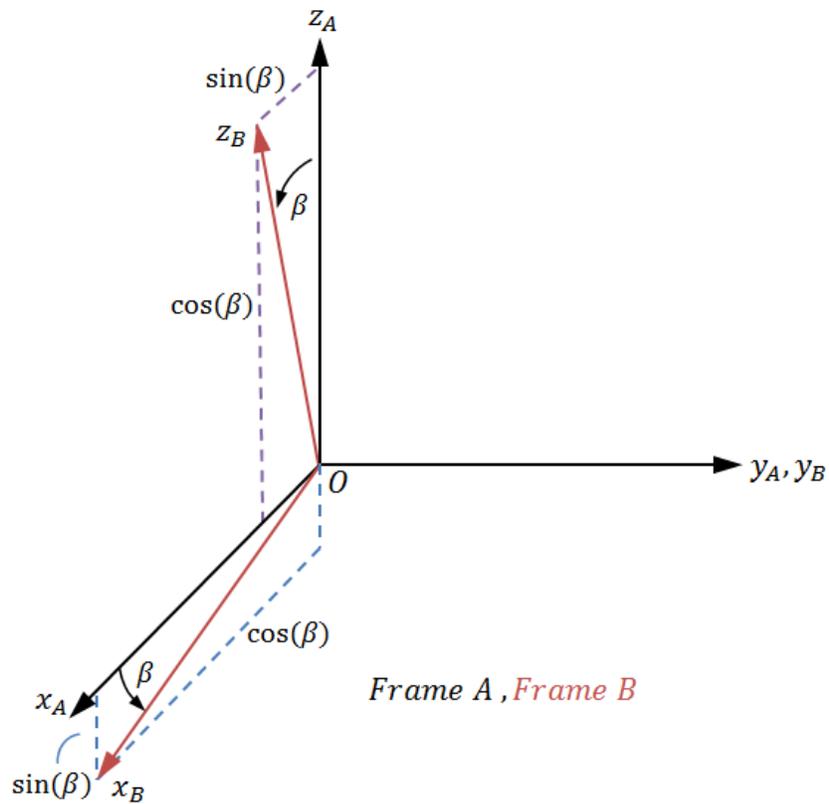
Fig. 6: Rotation about the y-axis

Rotation about the x-axis

If a reference frame (Frame A) is rotated by an angle γ about the x-axis to obtain a new frame (Frame B), the rotation matrix of the new frame is

$$R_{AB} = R_x(\gamma) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\gamma) & -\sin(\gamma) \\ 0 & \sin(\gamma) & \cos(\gamma) \end{bmatrix}$$

... Eq. (8)
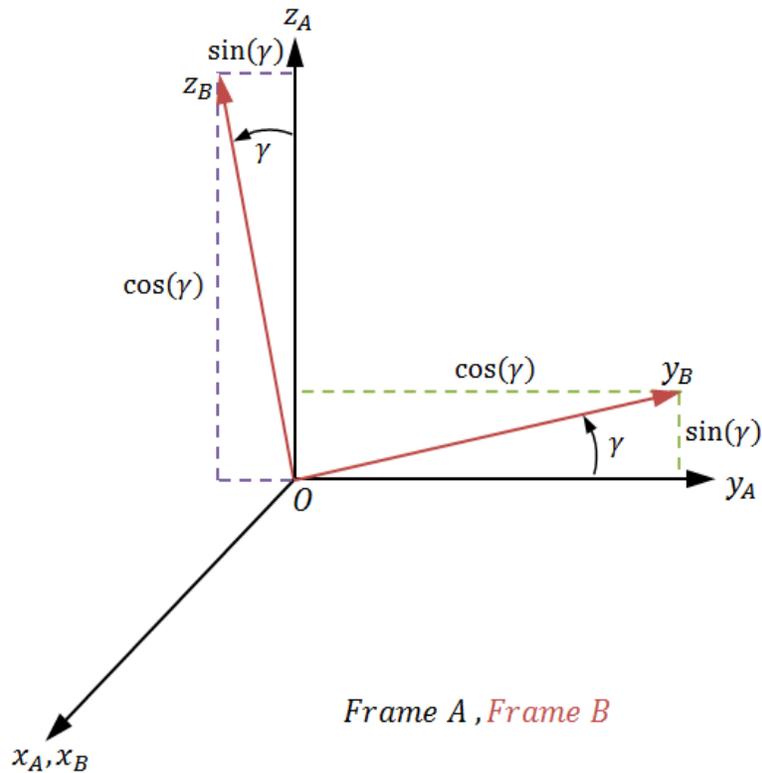
Fig. 7: Rotation about the-x axis

Cascade Rotations

The final orientation of three successive rotations made about moving axes (see Fig. 8) is the same as the final orientation of the three same rotations taken in the opposite order about fixed axes (see Fig. 9). Here, moving axes refers to the new axes

obtained after each rotation.
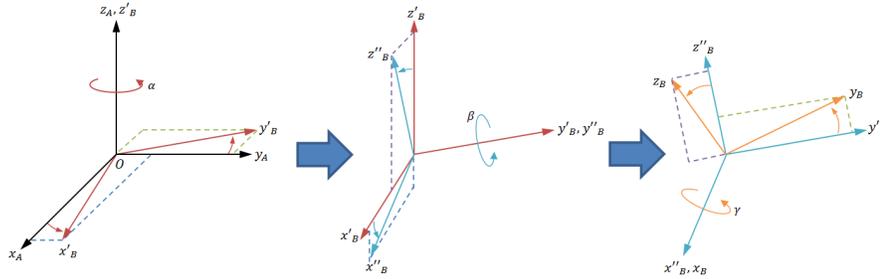
Fig. 8: Three rotations about moving axes
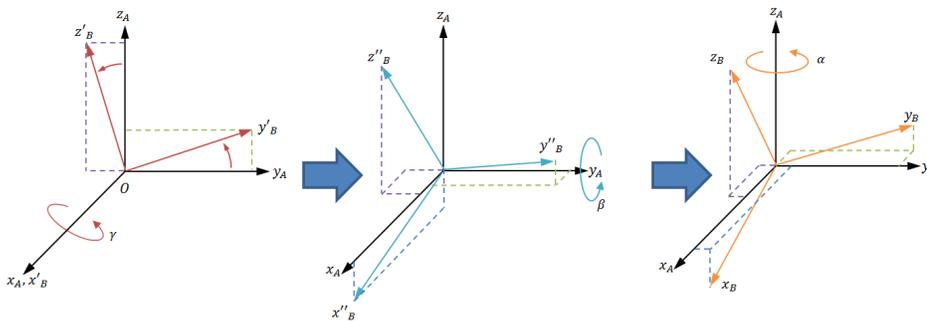
Fig. 9: Three rotations about fixed axes

Mathematically,

$$R_{AB,\,moving}(\alpha,\,\beta,\,\gamma) = R_{AB,\,fixed}(\gamma,\,\beta,\,\alpha) = R_z(\alpha) \cdot R_y(\beta) \cdot R_x(\gamma)$$

... Eq. (9)

The following video shows a comparison of the final orientations of a coordinate system after three successive rotations made about fixed and moving axes in the same order. The frame on the right is rotated about its moving axes and the frame on the left is rotated about fixed axes. The red axis corresponds to the x-axis, the green axis corresponds to the y-axis and the blue axis corresponds to the z-axis (order of rotations: 0.5 rad about z-axis, 0.75 rad about y-axis, 1 rad about x-axis).
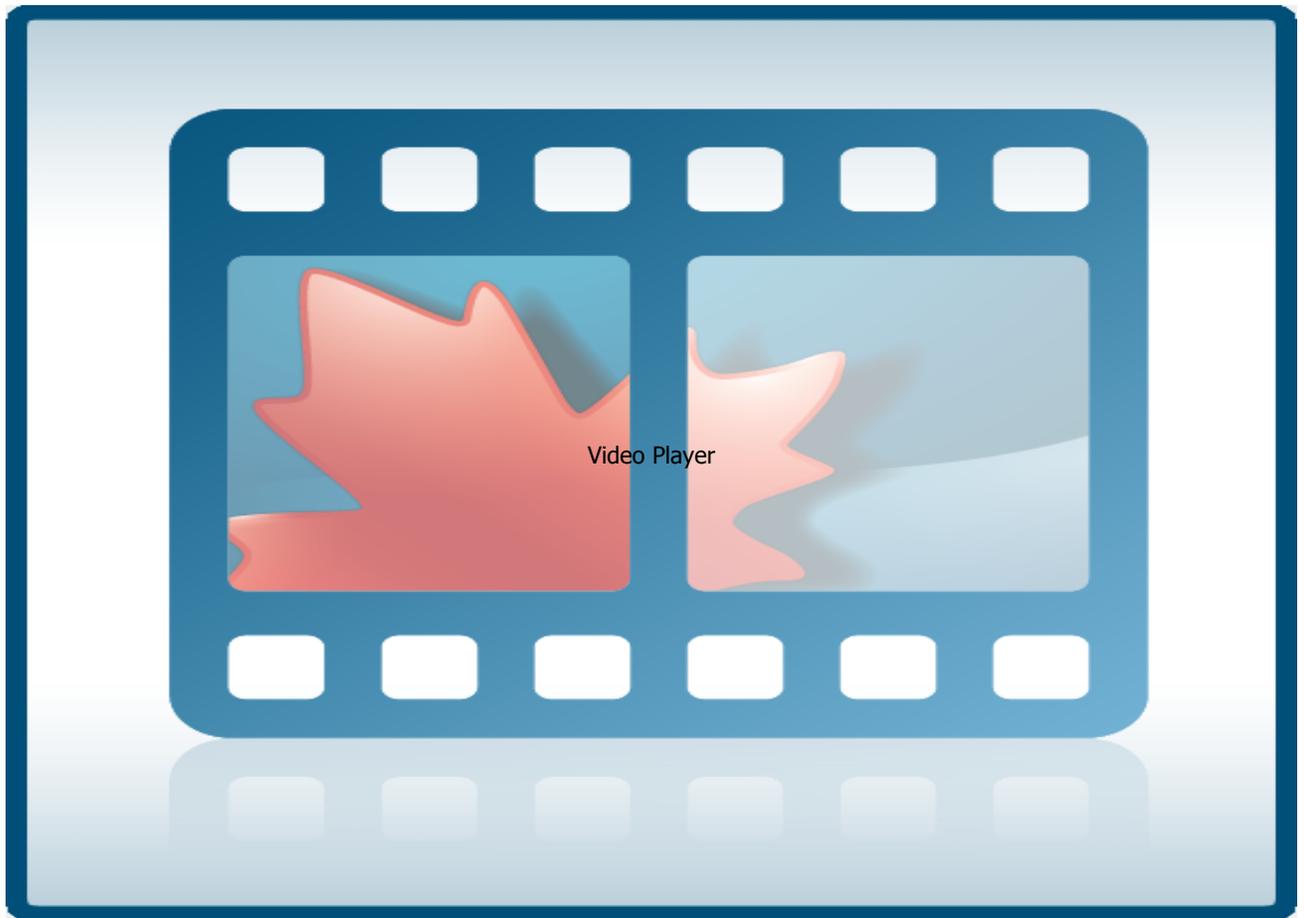
Video 1: Rotations about fixed and moving axes (some order).

As discussed above, since one frame rotates about its moving axes and the other rotates about fixed axes, the two final orientations are different.

The next video also shows a comparison of the final orientations of a coordinate system after three successive rotations made about fixed and moving axes. However, for this case, the order of rotations for the frame rotating about fixed axes is the opposite to the order for the frame rotating about its moving axes. The frame on the right is rotated about its moving axes and the frame on the left is rotated about fixed axes. Once again, the red axis corresponds to the x-axis, the green axis corresponds to the y-axis and the blue axis corresponds to the z-axis (order of rotations for the frame rotating about moving axes: 0.5 rad about z-axis, 0.75 rad about y-axis, 1 rad about x-axis).

Video Player
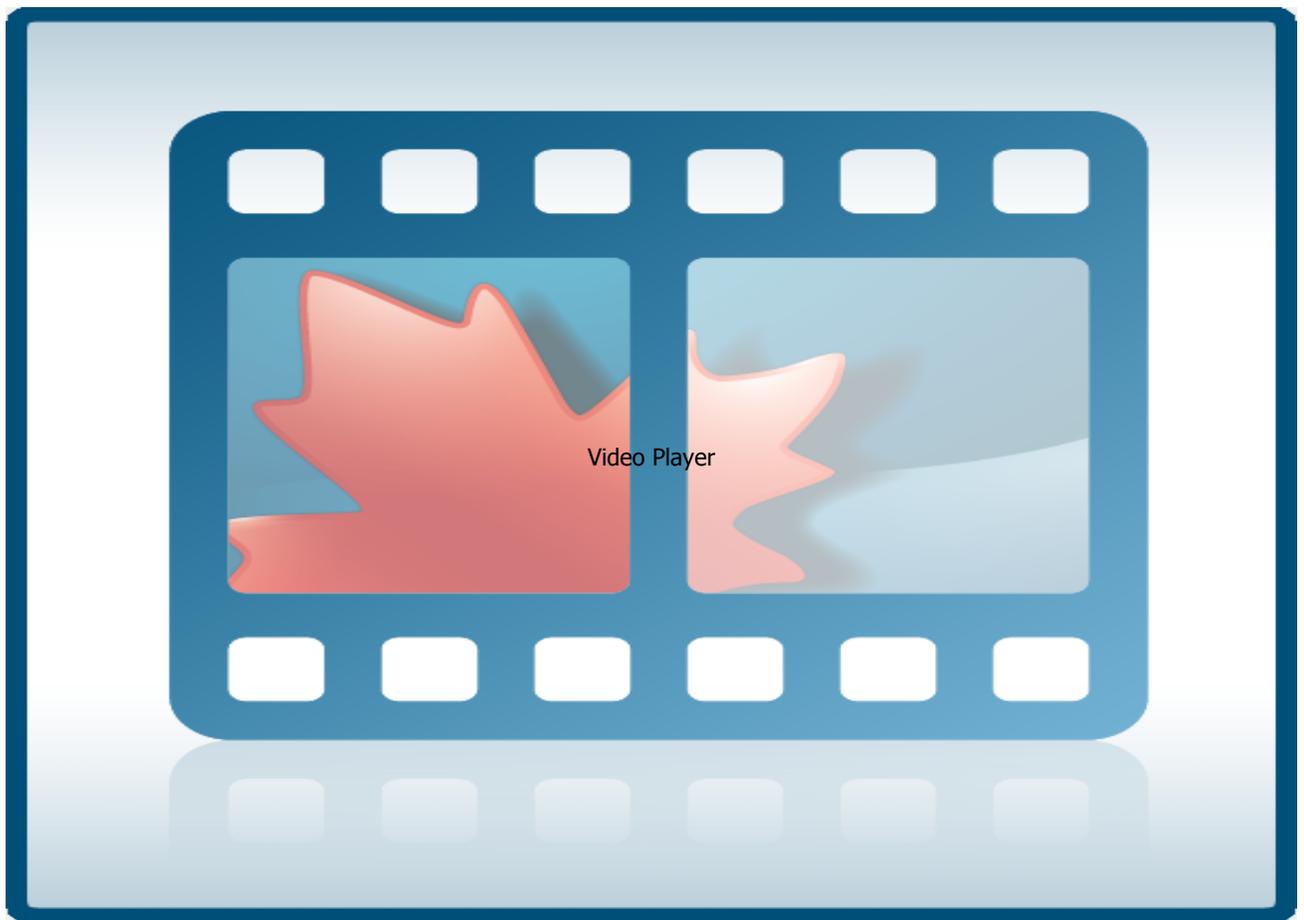
Video 1: Rotations about fixed and moving axes (some order).

For this case, the orientations of both frames are the same at the end.

▶ **Steps to create the simulations**

Euler Angle Representations

The Euler angle representations are commonly used representations that describe orientations. These representations describe an orientation using three successive rotations. Since rotation is a motion with three degrees of freedom, a set of three independent parameters are sufficient to describe an orientation in space.

Roll-Pitch-Yaw Angles

This representation describes an orientation using a set of three successive

rotations about a fixed frame.
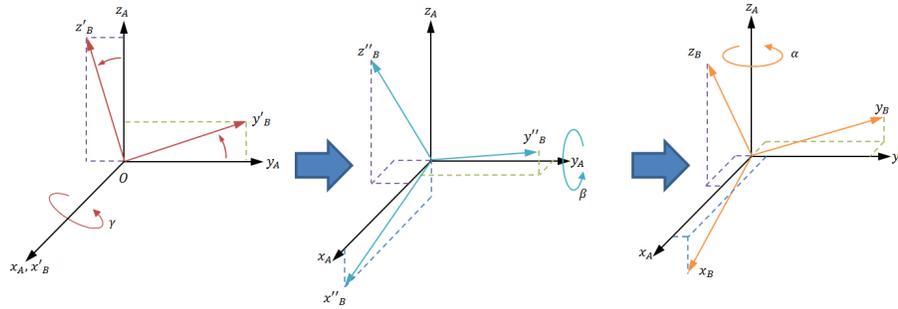
Fig. 14: Three rotations about fixed axes

The angle γ (rotation about the x-axis) is called the roll angle, the angle β (rotation about the y-axis) is called the pitch angle and the angle α (rotation about the z axis) is called the yaw angle.

The resulting rotation matrix of the three rotations is

$$
\begin{aligned}
R_z(\alpha) \cdot R_y(\beta) \cdot R_x(\gamma) = [\, &[\cos(\alpha)\ \cos(\beta),\ -\sin(\alpha)\ \cos(\gamma) + \cos(\alpha)\ \sin(\beta)\ \sin(\gamma),\ \sin(\alpha)\ \sin(\gamma) \\
&+ \cos(\alpha)\ \sin(\beta)\ \cos(\gamma)\,], \\
&[\sin(\alpha)\ \cos(\beta),\ \cos(\alpha)\ \cos(\gamma) + \sin(\alpha)\ \sin(\beta)\ \sin(\gamma),\ -\cos(\alpha)\ \sin(\gamma) \\
&+ \sin(\alpha)\ \sin(\beta)\ \cos(\gamma)\,], \\
&[\,-\sin(\beta),\ \cos(\beta)\ \sin(\gamma),\ \cos(\beta)\ \cos(\gamma)\,]\,]
\end{aligned}
$$

... Eq. (10)

Since the multiplication of matrices do not usually commute, the order of the rotations is important.

If a rotation matrix is given, the roll-pitch-yaw angles can be calculated using the following equations:

$$
\beta = \sin^{-1}(-a_{3,1})
$$
$$
\alpha = \tan^{-1}\left(\frac{a_{2,1}}{\cos(\beta)},\ \frac{a_{1,1}}{\cos(\beta)}\right)
$$
$$
\gamma = \tan^{-1}\left(\frac{a_{3,2}}{\cos(\beta)},\ \frac{a_{3,3}}{\cos(\beta)}\right)
$$

... Eqs. (11) to (13)

where $a_{i,j}$ corresponds to the term in the ith row and j^{th} column of the rotation

matrix.

z-y-z Euler Angles

This representation describes an orientation using a set of three successive rotations about moving axes.
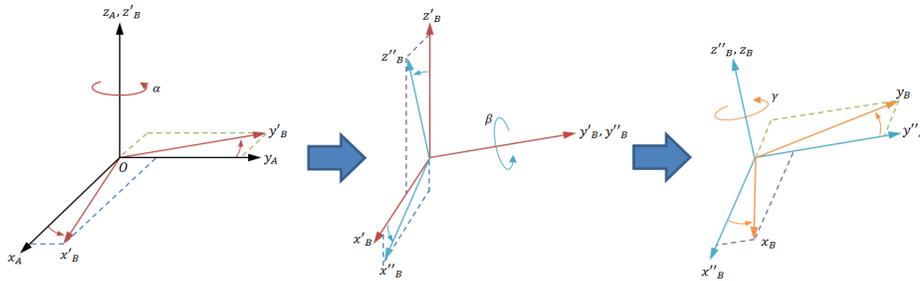
Fig. 15: Three rotations about moving axes (z-y-z angles)

The resulting rotation matrix of the three rotations is

$$
\begin{aligned}
R_z(\alpha) \cdot R_y(\beta) \cdot R_z(\gamma) = & \,[\,[\,-\sin(\alpha)\,\sin(\gamma) + \cos(\beta)\,\cos(\gamma)\,\cos(\alpha),\ -\sin(\alpha)\,\cos(\gamma) \\
& - \cos(\beta)\,\sin(\gamma)\,\cos(\alpha),\ \cos(\alpha)\,\sin(\beta)\,], \\
& [\,\cos(\alpha)\,\sin(\gamma) + \cos(\beta)\,\cos(\gamma)\,\sin(\alpha),\ \cos(\alpha)\,\cos(\gamma) - \cos(\beta)\,\sin(\gamma)\,\sin(\alpha), \\
& \sin(\alpha)\,\sin(\beta)\,], \\
& [\,-\cos(\gamma)\,\sin(\beta),\ \sin(\gamma)\,\sin(\beta),\ \cos(\beta)\,]\,]
\end{aligned}
$$

... Eq. (14)

If a rotation matrix is given, the z-y-z angles can be calculated using the following equations:

$$
\beta = \cos^{-1}(a_{3,3})
$$

$$
\alpha = \tan^{-1}\left(\frac{a_{2,3}}{\sin(\beta)},\ \frac{a_{1,3}}{\sin(\beta)} \right)
$$

$$
\gamma = \tan^{-1}\left(\frac{a_{3,2}}{\sin(\beta)},\ -\frac{a_{3,1}}{\sin(\beta)} \right)
$$

... Eqs. (15) to (17)

Example 1: Euler Angles

Problem Statement: The rotation matrix of Frame B with respect to a reference

frame A is $\begin{bmatrix} 0.0726 & -0.597 & 0.799 \\ 0.448 & 0.734 & 0.508 \\ -0.891 & 0.321 & 0.321 \end{bmatrix}$. Find the roll-pitch-yaw angles and the z-y-z

angles.

Solution:

The rotation matrix is:

$$R_{AB} := \begin{bmatrix} 0.0726 & -0.597 & 0.799 \\ 0.448 & 0.734 & 0.508 \\ -0.891 & 0.321 & 0.321 \end{bmatrix} :$$

The roll, pitch and yaw angles are (in rad):

$$\beta_{pitch} := \sin^{-1}\left(-(R_{AB})_{3, 1}\right)$$

$$1.099543058 \tag{2.2.4.3.1}$$

$$\alpha_{yaw} := \tan^{-1}\left(\frac{(R_{AB})_{2, 1}}{\cos(\beta_{pitch})}, \frac{(R_{AB})_{1, 1}}{\cos(\beta_{pitch})}\right)$$

$$1.410139396 \tag{2.2.4.3.2}$$

$$\gamma_{roll} := \tan^{-1}\left(\frac{(R_{AB})_{3, 2}}{\cos(\beta_{pitch})}, \frac{(R_{AB})_{3, 3}}{\cos(\beta_{pitch})}\right)$$

$$0.7853981634 \tag{2.2.4.3.3}$$

The z-y-z angles are (in rad):

$$\beta_y := \cos^{-1}\left((R_{AB})_{3, 3}\right)$$

$$1.244011150 \tag{2.2.4.3.4}$$

$$\alpha_z := \tan^{-1}\left(\frac{(R_{AB})_{2, 3}}{\sin(\beta_y)}, \frac{(R_{AB})_{1, 3}}{\sin(\beta_y)}\right)$$

$$0.5663241952 \tag{2.2.4.3.5}$$

$$\gamma_z := \tan^{-1}\left(\frac{\left(R_{AB}\right)_{3,\,2}}{\sin\left(\beta_y\right)}, -\frac{\left(R_{AB}\right)_{3,\,1}}{\sin\left(\beta_y\right)}\right)$$

<div align="center">0.3457940165</div>

<div align="right">**(2.2.4.3.6)**</div>

▼ Coordinate Transformations
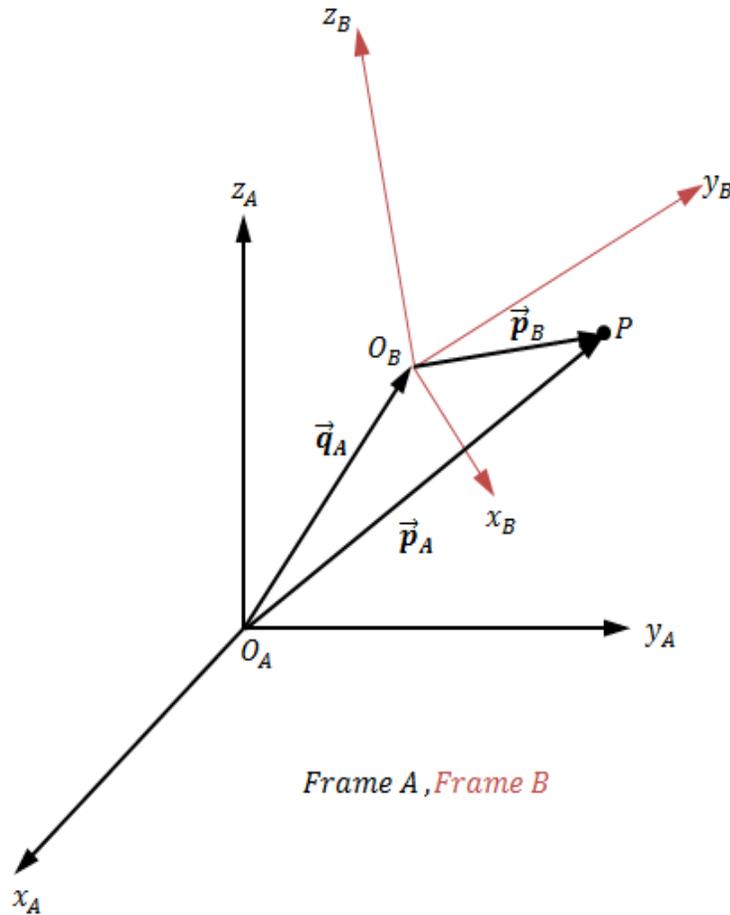
Fig. 16: Spatial displacement

Consider the point P in Fig. 16. Its position vector relative to Frame A can be found using the following vector sum:

$$\overrightarrow{O_A P} = \overrightarrow{O_A O_B} + \overrightarrow{O_B P}$$

<div align="right">... Eq. (18)</div>

This can be written as

$$\vec{p}_A = \vec{q}_A + R_{AB} \cdot \vec{p}_B$$

<div align="right">... Eq. (19)</div>

where R_{AB} is the rotation matrix of Frame B with respect to Frame A, $\vec{q}_A$ is the position vector of the origin O_B of Frame B with respect to Frame A and $\vec{p}_B$ is the position vector of point P with respect to Frame B. This is the general transformation of a position vector from one frame to another. To make this equation more compact, the concepts of homogeneous coordinates and homogeneous transformation matrix are introduced.

▼ Homogeneous Transformation Matrix

The homogeneous transformation matrix is a 4x4 matrix that is defined for mapping a position vector from one coordinate system to another. The matrix has the following form

$$T_{AB} = \begin{bmatrix} R_{AB} \left(3 \times 3 \right) & \vdots & \vec{q}_A \left(3 \times 1 \right) \\ \cdots & \vdots & \cdots \\ 0\,0\,0 & \vdots & 1 \end{bmatrix}$$

<div align="right">... Eq. (20)</div>

Using this matrix we can rewrite Eq. (19) as

$$\begin{bmatrix} \vec{p}_A \\ \text{---} \\ 1 \end{bmatrix}_{(4 \times 1)} = T_{AB} \cdot \begin{bmatrix} \vec{p}_B \\ \text{---} \\ 1 \end{bmatrix}_{(4 \times 1)}$$

<div align="right">... Eq. (21)</div>

Here $\begin{bmatrix} \vec{p}_A \\ \hline 1 \end{bmatrix}_{(4 \times 1)}$ and $\begin{bmatrix} \vec{p}_B \\ \hline 1 \end{bmatrix}_{(4 \times 1)}$ are 4 dimensional vectors that are called

homogeneous position vectors. They simply allow for the transformations to be written and computed in a compact form. The first three elements of a homogeneous position vector are the components of the corresponding position vector and the fourth element is 1. By introducing a new notation to represent homogeneous vectors, Eq. (21) can be written as

$$^A\boldsymbol{p} = T_{AB} \cdot {}^B\boldsymbol{p}$$

... Eq. (22)

Inverse of a Homogeneous Transformation Operator

$$\left(T_{AB}\right)^{-1} = T_{BA}$$

... Eq. (23)

Successive Transformations
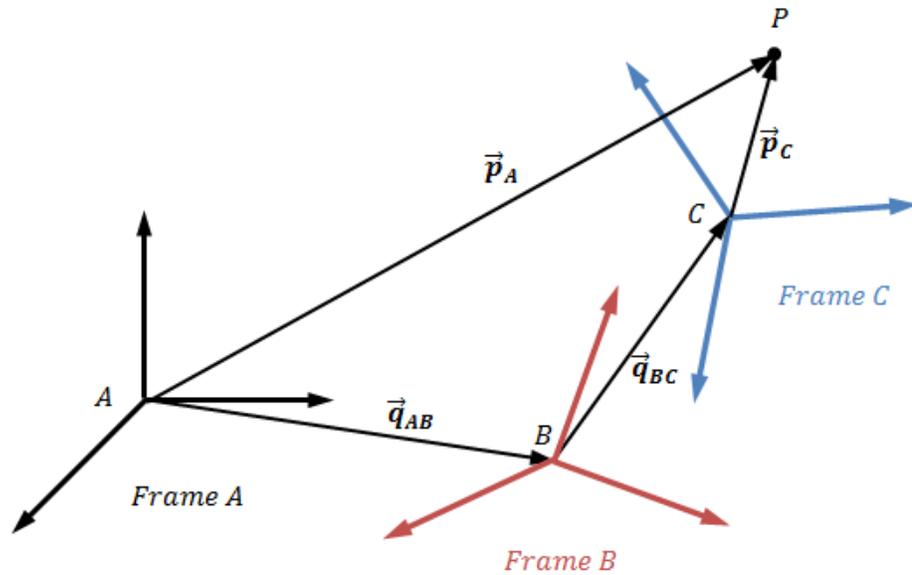
Fig. 17: Three reference frames

Consider the point P in Fig. (17). The homogeneous position vector of point P with respect of Frame C is ^Cp. To find the position vector of the point with respect to Frame B, the following transformation is required

$$^Bp = T_{BC} \cdot {}^Cp$$

... Eq. (24)

Similarly, to find the position vector of point P with respect to Frame C, the following transformations are required

$$^Ap = T_{AB} \cdot {}^Bp = T_{AB} \cdot T_{BC} \cdot {}^Cp$$

... Eq. (25)

This means that

$$T_{AC} = T_{AB} \cdot T_{BC}$$

... Eq. (26)

These steps show that multiplying the transformation matrices is equivalent to taking successive transformations. The following is the transformation matrix for two successive transformations.

$$T_{AC} = T_{AB} \cdot T_{BC} = \begin{bmatrix} R_{AB}R_{BC} & \vdots & R_{AB} \cdot \vec{q}_{BC} + \vec{q}_{AB} \\ \cdots & \vdots & \cdots \\ [0] & \vdots & 1 \end{bmatrix}$$

... Eq. (27)

Example 2: Robot Arm (with MapleSim)
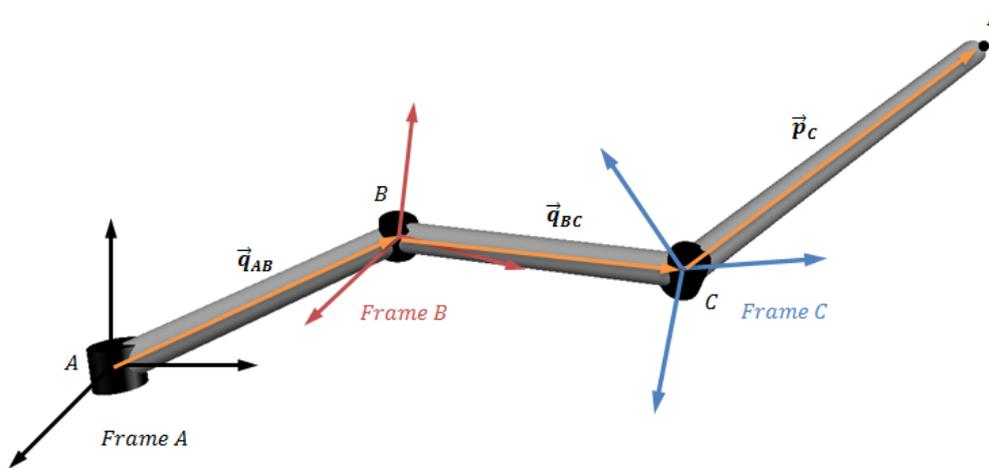
Fig. 18: Three segment robotic arm

Problem Statement: A robot arm (see Fig. 18), consisting of three segments, has end A fixed to the ground and end P free to perform tasks. The arm has three joints and each one has its own frame of reference. The orientation of Frame B with respect to Frame A is

$R_{AB} = \begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, the position vector of the origin of Frame B with respect to Frame A

is $\vec{q}_{AB} = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$, the orientation of Frame C with respect to Frame B is

$$R_{BC} = \begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.433 & 0.75 & -0.5 \\ 0.25 & 0.433 & 0.866 \end{bmatrix}$$, the position vector of the origin of Frame C with respect to

Frame B is $\vec{q}_{BC} = \begin{bmatrix} 0.5 \\ 0.25 \\ 0.75 \end{bmatrix}$ and the position vector of the point P with respect to Frame C is

$$\vec{p}_C = \begin{bmatrix} 0.5 \\ 1 \\ 1 \end{bmatrix}$$ (all lengths are in meters). Find the position vector of P with respect to Frame A.

▼ Analytical Solution

restart :
with(LinearAlgebra) :

To find the position vector of point P w.r.t Frame A, we have to transform the given position vector $\vec{p}_C$ from Frame C to Frame B and then from Frame B to Frame A. The first step is to write the transformation matrices:

$$T_{AB} := \begin{bmatrix} 0.866 & -0.5 & 0 & 0.5 \\ 0.5 & 0.866 & 0 & 0.5 \\ 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} :$$

and

$$T_{BC} := \begin{bmatrix} 0.866 & -0.5 & 0 & 0.5 \\ 0.433 & 0.75 & -0.5 & 0.25 \\ 0.25 & 0.433 & 0.866 & 0.75 \\ 0 & 0 & 0 & 1 \end{bmatrix} :$$

The homogeneous position vector of point P w.r.t Frame C is

$$p_C := \begin{bmatrix} 0.5 \\ 1 \\ 1 \\ 1 \end{bmatrix} :$$

Now, these three matrices can be multiplied to obtain the homogeneous position vector of point P w.r.t Frame A.

$$p_A := MatrixMatrixMultiply\big(T_{AB}, MatrixMatrixMultiply\big(T_{BC}, p_C\big)\big)$$

$$\begin{bmatrix} 0.516728000000000 \\ 1.33698900000000 \\ 2.67400000000000 \\ 1. \end{bmatrix}$$

(3.1.1)

Therefore, the position vector of point P with respect to Frame A is $\begin{bmatrix} 0.517 \\ 1.337 \\ 2.674 \end{bmatrix}$ (m).

Solution with MapleSim

Constructing the model

Step 1: Insert component

1. Drag the following components into the workspace:

Table 3: Components and locations

Component	Location
Fixed Frame	Multibody > Bodies and Frames
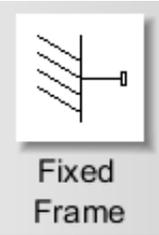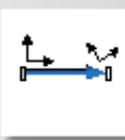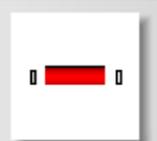
Rigid Body Frame (3 required)	Multibody > Bodies and Frames
Cylindrical Geometry (3 required)	Multibody > Visualization
Absolute Translation	Multibody > Sensors

2. Copy the **Axes** subsystem created in the Cascade Rotations subsection and paste it twice into the workspace (or follow the steps provided in that subsection to create the subsystem again).

Step 2: Connect components

Connect the subsystems as shown in the following diagram.
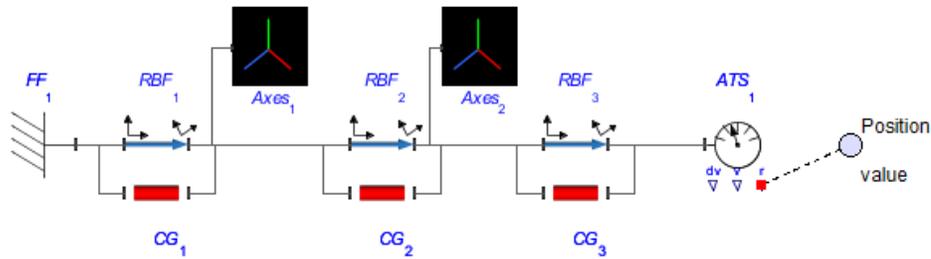
Fig. 19: Robot arm component diagram

Step 3: Set the component parameters

1. Click the **Rigid Body Frame** component connected to the **Fixed Frame** component and enter $\begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$ for the x,y,z offset ($\bar{r}_{XYZ}$).

2. Select **Rotation Matrix** in the drop-down menu for **Type$_R$** and enter

$\begin{bmatrix} 0.866 & 0.5 & 0 \\ -0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ for **[R]**. This is the transpose (and the inverse) of R_{AB}. A **Rigid Body Frame** component provides a frame of reference **(frame_b)** that has a fixed displacement ($\bar{r}_{XYZ}$) and a fixed orientation **([R])** relative to another frame of reference **(frame_a)**.

3. Click the next **Rigid Body Frame** component in the chain and enter $\begin{bmatrix} 0.5 \\ 0.25 \\ 0.75 \end{bmatrix}$ for the x,y,z offset ($\bar{r}_{XYZ}$).

4. Select **Rotation Matrix** in the drop-down menu for **Type$_R$** and enter

$\begin{bmatrix} 0.866 & 0.433 & 0.25 \\ -0.5 & 0.75 & 0.433 \\ 0 & -0.5 & 0.866 \end{bmatrix}$ for **[R]**. This is the transpose (and the inverse) of R_{BC}.

5. Click the third **Rigid Body Component** and enter

5. $\begin{bmatrix} 0.5 \\ 1 \\ 1 \end{bmatrix}$ for the x,y,z offset ($\overline{r}_{XYZ}$).

6. Click the **Absolute Translation** sensor component and select **Inertial** in the **Frame** drop-down menu. Frame A is the ground frame and coincides with the inertial frame (to measure the position of the point P relative to another frame a **Relative Translation** sensor component can be used).

Step 4: Run the Simulation

1. Attach a **Probe** to the **r** port of the **Absolute Translation Sensor** component.
2. Click the **Probe** and select **1**, **2**, and **3** in the **Inspector Pane**.
3. Change the **Simulation duration (t_d)** in the **Settings Pane** to **0.1** s or any other small value. Since there is no motion involved the duration of the simulation is not important.
4. Click **Run Simulation** (▶).

This simulation outputs three plots that give the position of the point P with respect to Frame A (see Fig. 20). Since the model does not move, the plots will show horizontal lines. The results of these plots match the results obtained analytically.
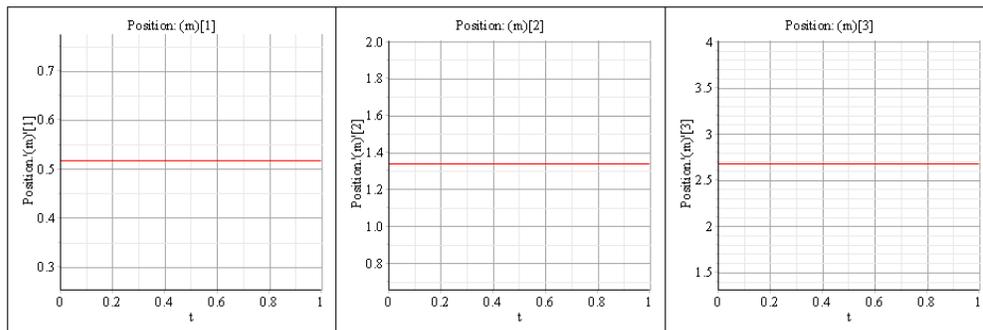
Fig. 20: Simulation results

Fig. 21 shows the 3-D visualization of the robot arm. The three sets of axes correspond

to Frames A, B and C.

Playback mode
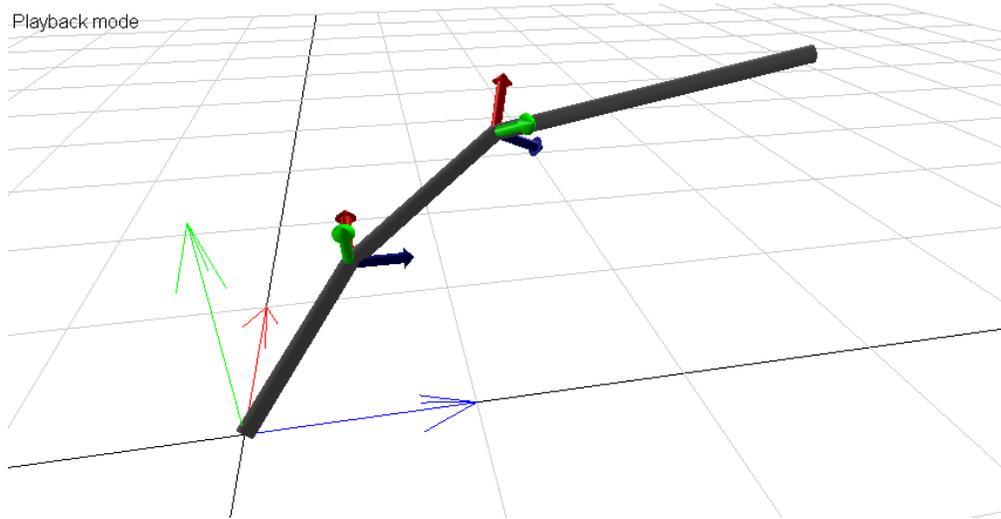
Fig. 21: 3-D visualization

References:
1. L-W. Tsai. "Robot Analysis: The Mechanics of Serial and Parallel Manipulators". NY, 1999, John Wiley & Sons, Inc.
2. W. W. Melek. "ME 547: Robot Manipulators: Kinematics, Dynamics, and Control". Waterloo, ON, 2010, University of Waterloo.